Radiation and Chemical Reaction Effects on MHD Flow of Viscoelastic Fluid with Mass Transfer

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Abstract

This paper is focused on the effects of radiation, heat and mass transfer on a unsteady free convective viscoelastic fluid flow of incompressible, electrically conducting and chemically reacting fluid past an impulsively started moving vertical plate adjacent to Darcian porous regime in the presence of heat generation. The dimensionless governing equations are solved using perturbation technique. A parametric study is performed to illustrate the influence of thermophysical parameters on the velocity, temperature and concentration profiles are presented graphically.

Keywords -: Free convection, Mass transfer, MHD, Porous medium, Viscoelastic fluid

1. Introduction

The role of thermal radiation is a major importance in some industrial applications such as glass production and furnace design and in space technology applications, such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and space craft re-entry aerothermodynamics which operate at high temperatures. In the processes involving high temperatures, the radiation heat transfer in combination with conduction, convection and also mass transfer plays very important role in the design of pertinent equipments in the areas such as nuclear power plants, gas turbines and the various propulsion devices for air craft, missiles, satellites and space vehicles. Chamkha et al. [3] studied the radiation effects on the free convection flow past a semi-infinite vertical plate with mass transfer. Chaudhary et al. [4] studied the radiation effects with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface. Emad M. Aboeldahad and Gamal El-Din A Azzam [7] investigated the thermal radiation effects on MHD flow past a semi-infinite vertical plate in the presence of mass diffusion. Ramachandra Prasad et al. [13] studied the transient radiative hydromagnetic free convection flow past an impulsively started vertical plate with uniform heat and mass flux. Ramachandra Prasad et al. [14] considered the radiation and mass transfer effects on a two-dimensional flow past an impulsively started isothermal vertical plate. The interaction of radiation with hydromagnetic flow has become industrially more prominent in the processes wherever high temperatures occur. Takhar et al. [16] analyzed the radiation effects on MHD free convection flow past a semi-infinite vertical plate using Runge-Kutta Merson quadrature**.** Abd-El-Naby et al. [1] studied the radiation effects on MHD unsteady free convection flow over a vertical plate with variable surface temperature**.**

The problem of free-convection and mass transfer flow of an electrically-conducting fluid past an infinite plate under the influence of a magnetic field has attracted interest in view of its application to geophysics, astrophysics, engineering, and to the boundary layer control in the field of aerodynamics. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [9]. Muthucumaraswamy et al. [11] studied heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature. On the other hand, considerable interest has been developed in the study of the interaction between magnetic fields and the flow of electrically-conducting incompressible elasto-viscous fluid due to its wide applications in modern technology. The study of an elasto-viscous pulsatile flow helps to understand the mechanisms of dialysis of blood through an artificial kidney. Rajesh and Varma [12] studied the effects of thermal radiation on unsteady free convection flow of an elasto-viscous fluid over a moving vertical plate with variable temperature in the presence of magnetic field through porous medium. Deka et al. [6] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [15]. Ch Kesavaiah et.al [10] effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Gireesh Kumar and Satyanarayana [8] Mass transfer effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Srinathuni Lavanya and Chenna Kesavaiah [17] studied Heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction, Bhavana et.al [18] analyzed the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source.

However, the interaction of radiation with heat and mass transfer in a chemically reacting and electrically conducting viscoelastic fluid past an impulsively started plate embedded in a Darcy porous medium in the presence of heat generation has received little attention. Hence, the present study is attempted. Such study has significant applications in solar collection systems, fire dynamics in insulations, and also geothermal energy systems. The volumetric heat generation term may exert a strong influence on the heat transfer and as a consequence, also on the fluid flow. The transformed problem is shown to be dictated by the thermo physical and hydrodynamic parameters, viz., dimensionless time, thermal Grashof number, species Grashof number, magnetic parameter, Darcy number, Reynolds number, Prandtl number, heat generation parameter, radiation parameter and Schmidt number. The influence of these parameters on the velocity profiles, temperature function, mass transfer function, local and average shear stresses, local and average Nusselt numbers and local and average Sherwood numbers are presented and discussed at length.

2. Formulation of the problem

The unsteady free convection and mass transfer flow of an electrically conducting incompressible elasto-viscous fluid past an infinite vertical plate through porous medium in the presence of radiating heat source in the presence of chemical reaction has been considered. A magnetic field of uniform strength B_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The flow is assumed to be in $x-$direction which is taken along the vertical plate in the upward direction .The $y-$axis is taken to be normal to the plate. Initially the plate and the fluid are at the same temperature *T'* with concentration level C'_∞ at all points. At time $t'>0$, the plate is exponentially accelerated with a velocity $u=\varepsilon\left(\exp a't'\right)$ in its own plane and the plate temperature is raised linearly with time t and the level of concentration near the plate is raised to C'_w. The effect of viscous dissipation is assumed to be negligible. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'}=g\beta\left(T'-T'_\infty\right)+g\beta^*\left(C'-C'_\infty\right)+\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{K_0}{\rho}\left(\frac{\partial^3 u'}{\partial y'^2\partial t'}\right)-\frac{\nu}{K'}u'-\frac{\sigma B_0{}^2}{\rho}u' \tag{1}$$

$$\rho C_p\frac{\partial T'}{\partial t'}=K\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q_r}{\partial y'}+Q' \tag{2}$$

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2}-Kr'\left(C'-C'_\infty\right) \tag{3}$$

The initial and boundary conditions for the velocity, temperature and concentration fields are

$$u' = 0, T' = T'_\infty, C' = C'_\infty \quad for\ all \quad y', t' \le 0$$
$$u' = \varepsilon \exp(a't'), T' \to T'_\infty + (T'_w - T'_\infty) At', C' = C'_\infty \ , t' > 0\} \qquad at \quad y' = 0 \quad (4)$$
$$u' = 0, T' \to T'_\infty, C' \to C'_\infty \qquad as \ \ y' \to \infty$$

Where u' is the velocity of the fluid along the plate in the x' - direction, t' is the time, g is the acceleration due to gravity, β is the coefficient of volume expansion, β^* is the coefficient of thermal expansion with concentration, T'_∞ is the temperature of the fluid near the plate, T'_w is the temperature of the fluid far away from the plate, T'_w is the temperature of the fluid, C' is the species concentration in the fluid near the plate, C'_∞ is the species concentration in the fluid far away from the plate, ν is the kinematic viscosity, K_0 is the coefficient of kinematic visco-elastic parameter, σ is the electrical conductivity of the fluid, B_0 is the strength of applied magnetic field, ρ is the density of the fluid, C_p is the specific heat at constant pressure, K is the thermal conductivity of the fluid, μ is the viscosity of the fluid, D is the molecular diffusivity, u_0 is the velocity of the plate.

The radiative heat flux q_r is given by equation (5) in the sprit of Cogly et. al [5]

$$\frac{\partial q_r}{\partial y'} = 4(T' - T'_\infty) I \qquad (5)$$

where $I = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda$, $K_{\lambda w} -$ is the absorption coefficient at the wall and $e_{b\lambda} -$ is Planck's function, I is absorption coefficient

Equations (1) - (3) can be made dimensionless by introducing the following dimensionless variables and parameters:

In order to write the governing equations and the boundary conditions in dimensionless from, the following non-dimensional quantities are introduced.

$$u = \frac{u'}{u_0}, y = \frac{u_0 y'}{\nu}, t = \frac{t' u_0^2}{\nu}, \theta = \frac{T' - T_\infty'}{T'_w - T_\infty'}, K = \frac{K' u_0^2}{\nu^2}, Kr = \frac{Kr'\nu}{U_0^2}, a = \frac{a'\nu}{u_0^2}$$
$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \Pr = \frac{\mu C_p}{\kappa}, Q = \frac{\nu Q'}{\rho C_p u_0^2}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, S = \frac{K_0 u_0^2}{\rho \nu^2}, Sc = \frac{\nu}{D} \qquad (6)$$
$$R = \frac{4\nu I}{\rho C_p u_0^2}, \qquad Gr = \frac{\nu \beta g (T'_w - T'_\infty)}{u_0^3}, \qquad Gm = \frac{\nu \beta^* g (C'_w - C'_\infty)}{u_0^3}$$

where Gr is the thermal Grashof number, Gc is modified Grashof Number, $\Pr$ is Prandtl Number, M is the magnetic field, R is the radiation parameter, Sc is Schmdit number, Kr is Chemical Reaction, K is Porous Permeability, Q is Heat source parameter respectively.

In terms of the above dimensionless quantities,

Equations (1) - (2) reduces to

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-S\left(\frac{\partial^3 u}{\partial y^2 \partial t}\right)-M\,u-\frac{1}{K}u+Gr\,\theta+Gm\,C \qquad (7)$$

$$\Pr\frac{\partial \theta}{\partial t}=\frac{\partial^2 \theta}{\partial y^2}-(R-Q)\Pr\theta \qquad (8)$$

$$Sc\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial y^2}-Kr\,Sc\,C \qquad (9)$$

The corresponding boundary conditions are

$$\begin{aligned} &u=0,\theta=0,C=0 \qquad t\le 0 \quad for\ all\ \ y\\ &u=\exp(at),\theta=1,C=1,t>0 \quad at \quad y=0\\ &u=0,\theta\to 0,C\to 0 \qquad as\ \ y\to\infty \end{aligned} \qquad (10)$$

In the present analysis we have considered the heat generation (absorption) of the type

$$Q'=Q_0\left(T'-T'_\infty\right)$$

Where $\frac{Q'}{\rho C_p}$ is the volumetric rate of heat generation (absorption). For solving the problem, we take, Beard and Walters [2], u in the form

$$u=U_o+SU_1$$

3. Solution of the problem

Equation (7) – (9) are coupled, non – linear partial differential equations and these cannot be solved in closed – form using the initial and boundary conditions (10). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the fluid in the neighbourhood of the plate as

$$\begin{aligned} u&=u_0(y)+\varepsilon e^{at}u_1(y)\\ \theta&=\theta_0(y)+\varepsilon e^{at}\theta_1(y)\\ C&=C_0(y)+\varepsilon e^{at}C_1(y) \end{aligned} \qquad (11)$$

Substituting (11) in Equation (7) – (9) and equating the harmonic and non – harmonic terms, we obtain

$$u_0'' - N_1 u_0 = -Gr\,\theta_0 - Gm\,C_0 \tag{12}$$

$$\beta_4 u_1'' + \beta_5 u_1 = -Gr\,\theta_1 - Gm\,C_1 \tag{13}$$

$$\theta_0'' - \Pr\left(R - Q\right)\theta_0 = 0 \tag{14}$$

$$\theta_1'' - \beta_2\,\theta_1 = 0 \tag{15}$$

$$C_0'' - Sc\,Kr\,C_0 = 0 \tag{16}$$

$$C_1'' - \beta_1 C_1 = 0 \tag{17}$$

The corresponding boundary conditions can be written as

$$\left.\begin{array}{l} u_0 = 0, u_1 = 1,\ \theta_0 = 1 \\ \theta_1 = 0, C_0 = 1, C_1 = 0 \end{array}\right\} \quad at\ \ y = 0$$
$$\left.\begin{array}{l} u_0 \to 0, u_1 \to 0,\ \theta_0 \to 0 \\ \theta_1 \to 0, C_0 \to 0, C_1 \to 0 \end{array}\right\} \quad as\ \ y \to \infty \tag{18}$$

Solving Equations (12) - (17) under the boundary conditions (18) and we obtain the velocity, temperature and concentration distributions in the boundary layer as

$$u(y,t) = A_1 e^{m_6 y} + A_2 e^{m_2 y} + A_3 e^{m_{10} y} + \varepsilon e^{at}\left\{A_4 e^{m_8 y} + A_5 e^{m_{12} y}\right\}$$

$$\theta(y,t) = e^{m_6 y} + \varepsilon e^{at}\left\{e^{m_8 y}\right\}$$

$$C(y,t) = e^{m_2 y}$$

We now calculate Skin-friction from the velocity field. It is given in non-dimensional form as:

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = A_1 m_6 + A_2 m_2 + A_3\,m_{10} + \varepsilon e^{at}\left\{A_4\,m_8 + A_5\,m_{12}\right\}$$

where $\tau = -\dfrac{\tau'}{\rho {U_0}^2}$

The dimensionless rate of heat transfer is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = m_6 + \varepsilon m_8 e^{at}$$

The dimensionless Sherwood number is given by

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = m_2$$

4. Results and discussion

We have plotted velocity profiles, temperature and concentration for different parameters involving the problem i.e. Magnetic parameter (M), Permeability parameter(K), Viscoelastic parameter (S), mass Grashof number (Gm), time (t), Accelerating parameter (a), Heat source parameter (Q), Radiation parameter (R), Chemical reaction parameter (Kr) and Schmidt number (Sc) in Figures (1) to (13). The graphical presentation for distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters. In the present study we have chosen $\varepsilon = 0.02, t = 0.2$.

Figure (1) represents the velocity profiles for different values of accelerating parameter. From the figure the velocity is found to increase with an increase in accelerating parameter. It is also found that the fluid velocity due to the impulsive start of the plate (accelerating parameter is equal to zero) is less than due to the exponentially accelerated start (accelerating parameter is not equal to zero). Figures (2) illustrate the influences of magnetic parameter on the velocity field respectively. From this figure we found that the velocity increases with an increase in magnetic parameter. Figure (3) illustrates the velocity profiles for the different values of radiation parameter.

Figure (3) at a particular value of radiation parameter, the velocity and the thermal boundary layer thickness increase by increasing the angle of inclination, with an accompanying decrease in the wall velocity gradient. This is because of the reduction in the buoyancy force as the plate is inclined from the vertical to a large angular position. The velocity profiles observed from figure (4) for different values of permeability parameter. The velocity increase with an increase in permeability parameter. This is due to the fact that the presence of a porous medium increases the resistance to flow.

Figure (5) reveal velocity variations with mass Grashof number. From the figure it is observed that the velocity increases with an increase in mass Grashof number. It is due to the fact increase in the values of mass Grashof number has the tendency to increase the mass buoyancy effect. This gives rise to an increase in the induced flow. To observe the effect of heat source parameter, the velocity profile for different values of heat source parameter are presented in figures (6), the velocity increases near the surface of the plate and becomes maximum and then decreases away from the plate.

Figure (7) displays the effects of Schmidt number on the velocity field; it is found that the velocity increases with an increase in Schmidt number. Figure (8) display the effects of viscoelastic parameter on the velocity profiles, it is observed that the velocity is less for Newtonian fluid (viscoelastic parameter is equal to zero) than the Non-Newtonian fluid (viscoelastic parameter is not equal to zero) and also the velocity increases with an increase in viscoelastic parameter. The effects of radiation parameter on the temperature profiles are presented in Figure (9). From this figure we observe that, as the value of radiation parameter increases the temperature profiles decreases, with an increasing in the thermal boundary layer thickness.

Figure (10) shows the variation of temperature profiles for different values of heat source parameter. It is seen from this figure that temperature profiles increase with an increasing of heat generation parameter. Typical variation of the temperature profiles along the spanwise coordinate y are shown in figure (11) for different values of Prandtl number. The results show that an increase of Prandtl number results in a increasing the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. The reason is that smaller values of Prandtl number are equivalent to increasing the thermal conductivities, and therefore, heat is able to differ away from the heated surface more rapidly than for higher values of Prandtl number. Hence, the boundary layer is thicker and the rate of heat transfer is reduced, for gradient have been reduced.

For different values of the chemical reaction parameter, the concentration profiles plotted in figure (12). It is obvious that the influence of increasing values of chemical reaction parameter, the concentration distribution across the boundary layer decreases. Figure (13) shows the concentration profiles across the boundary layer for various values of Schmidt number. The figure shows that an increasing in Schmidt number results in a decreasing the concentration distribution, because the smaller values of Sc are equivalent to increasing the chemical molecular diffusivity.

5. References

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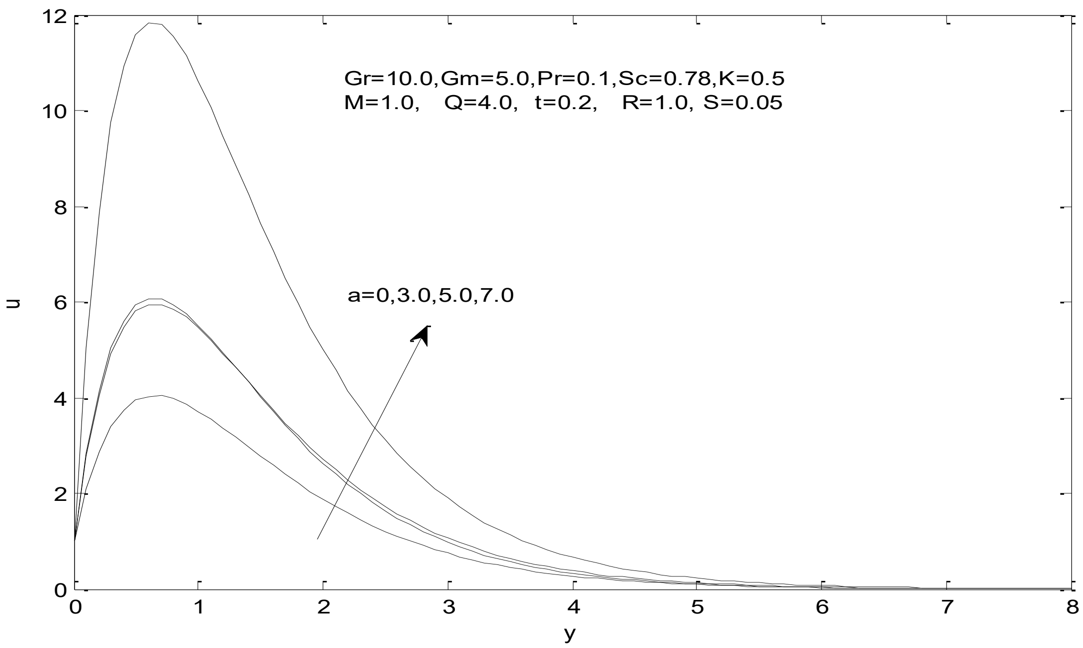

Figure (1): Velocity profiles for a

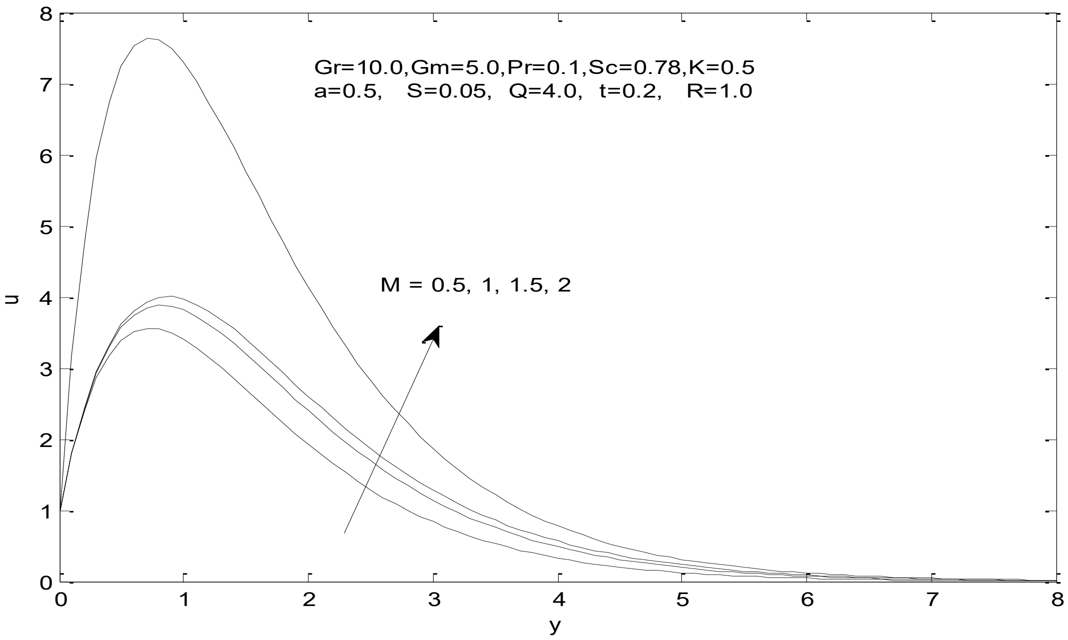

Figure (2): Velocity profiles for M

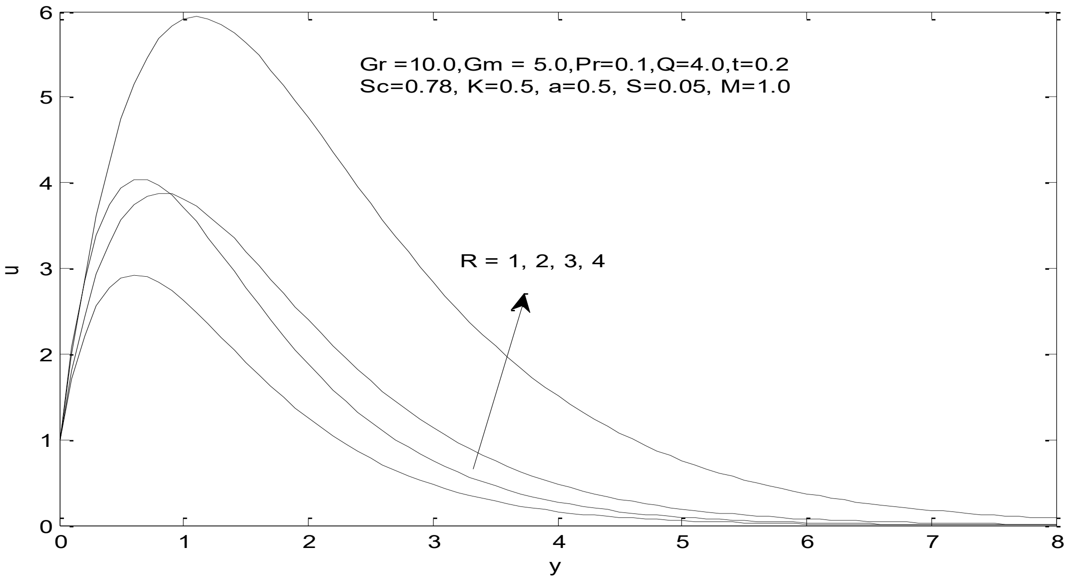

Figure (3): Velocity profiles for R

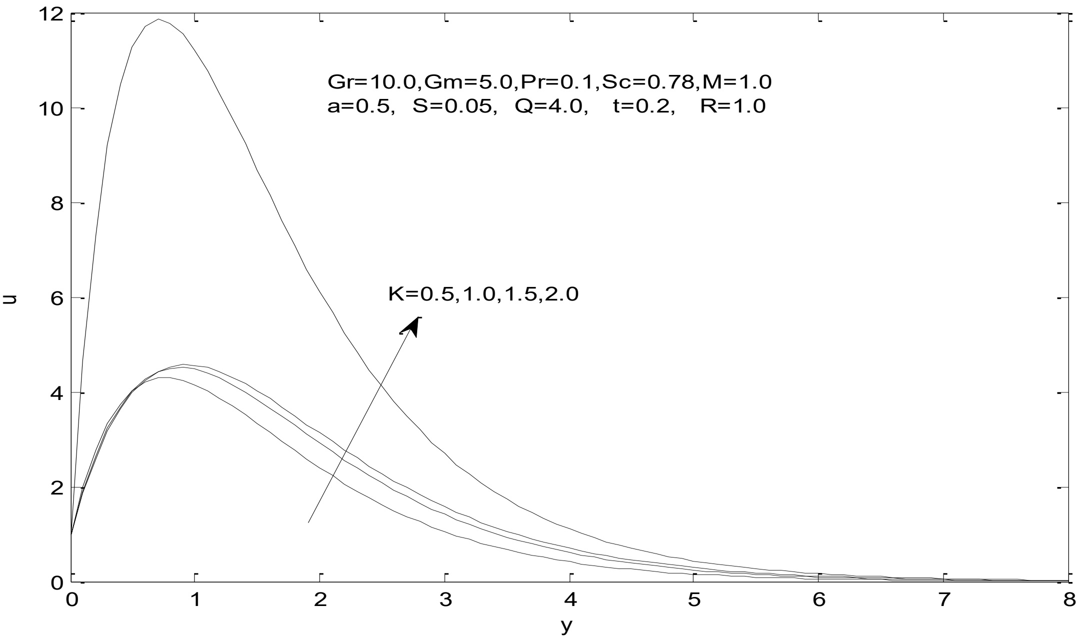

Figure (4): Velocity profiles for K

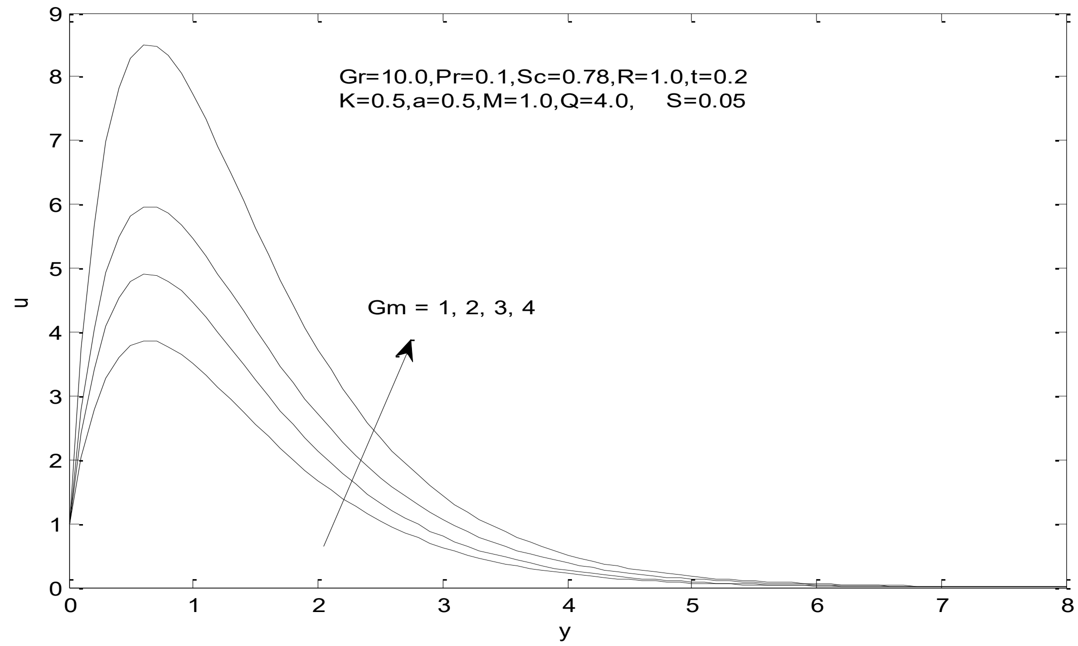

Figure (5): Velocity profiles for Gm

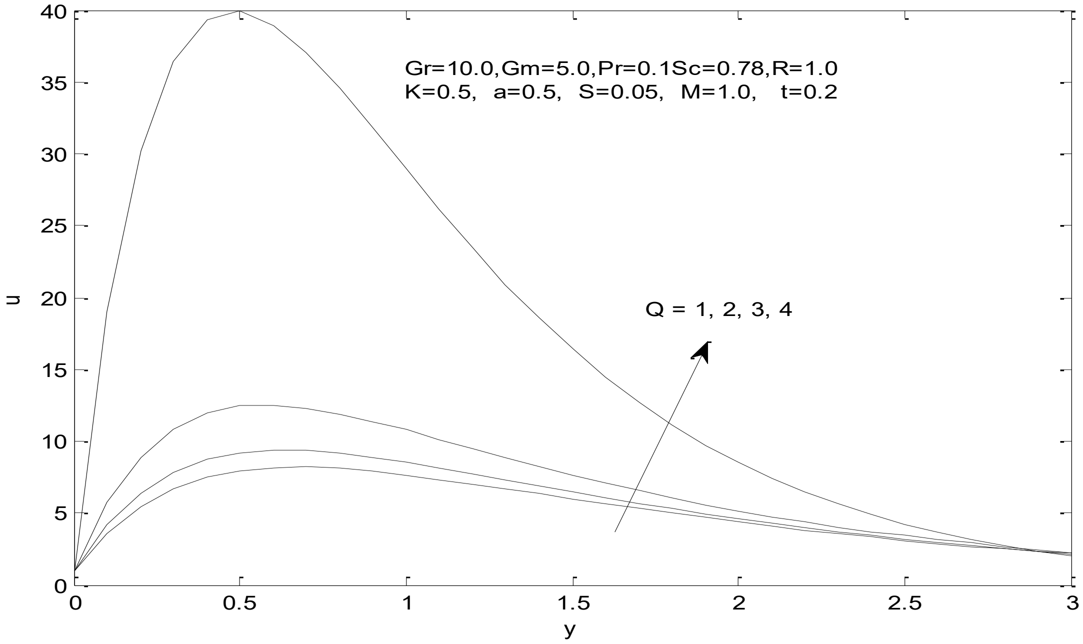

Figure (6): Velocity profiles for Q

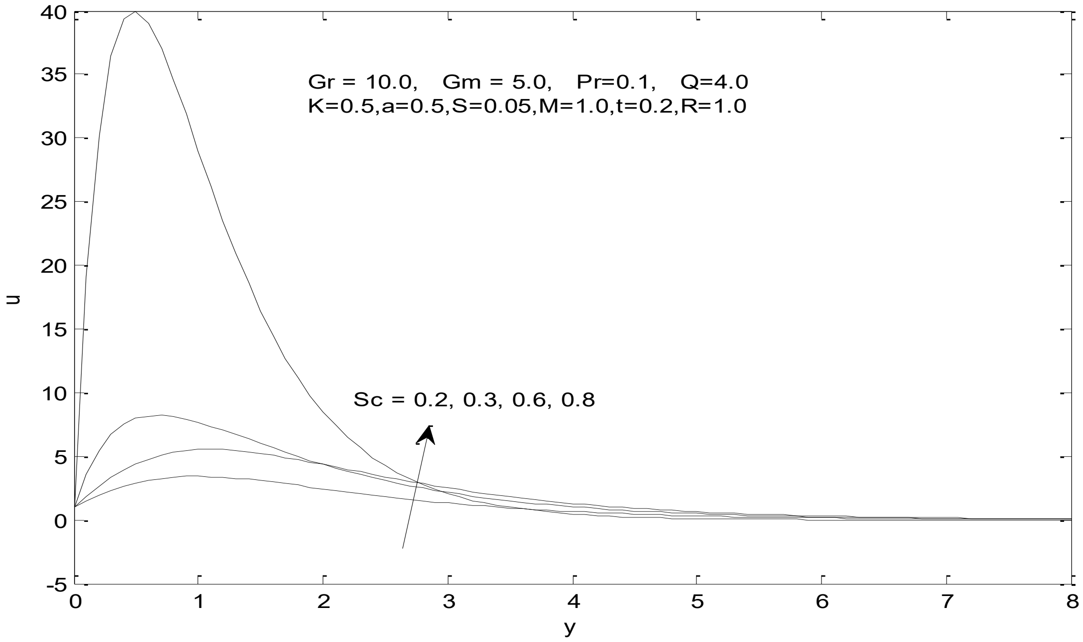

Figure (7): Velocity profiles for Sc

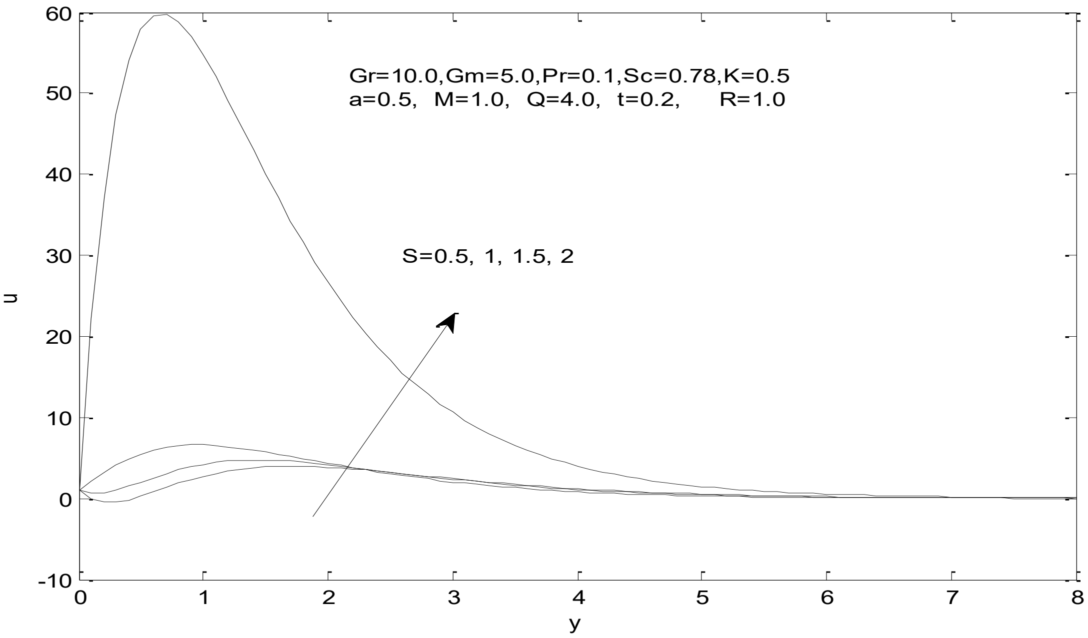

Figure (8) Velocity profiles for S

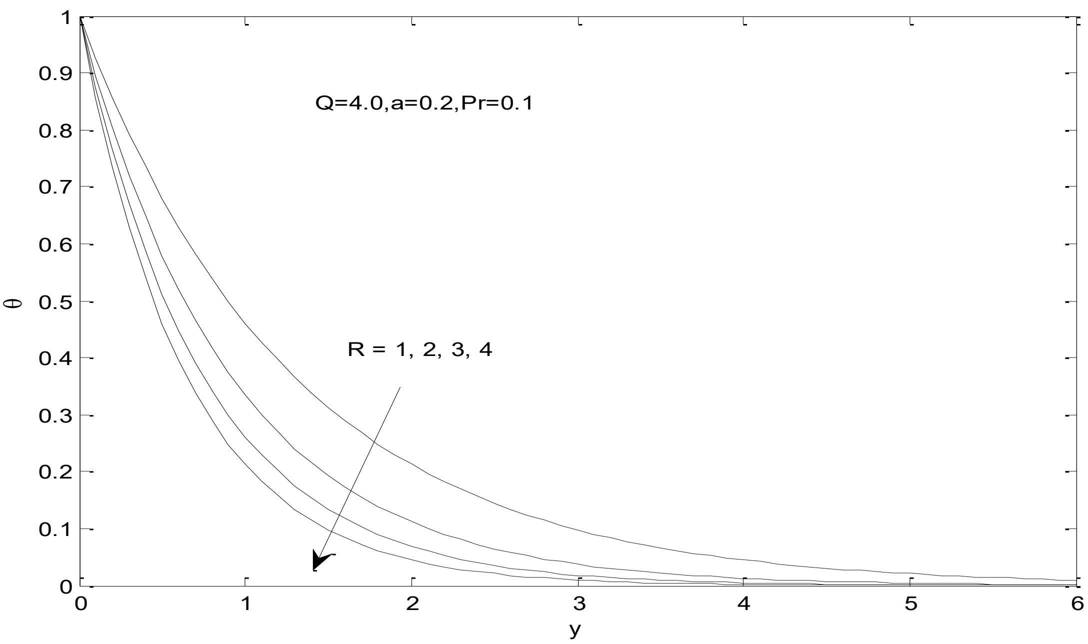

Figure (9): Temperature profiles for R

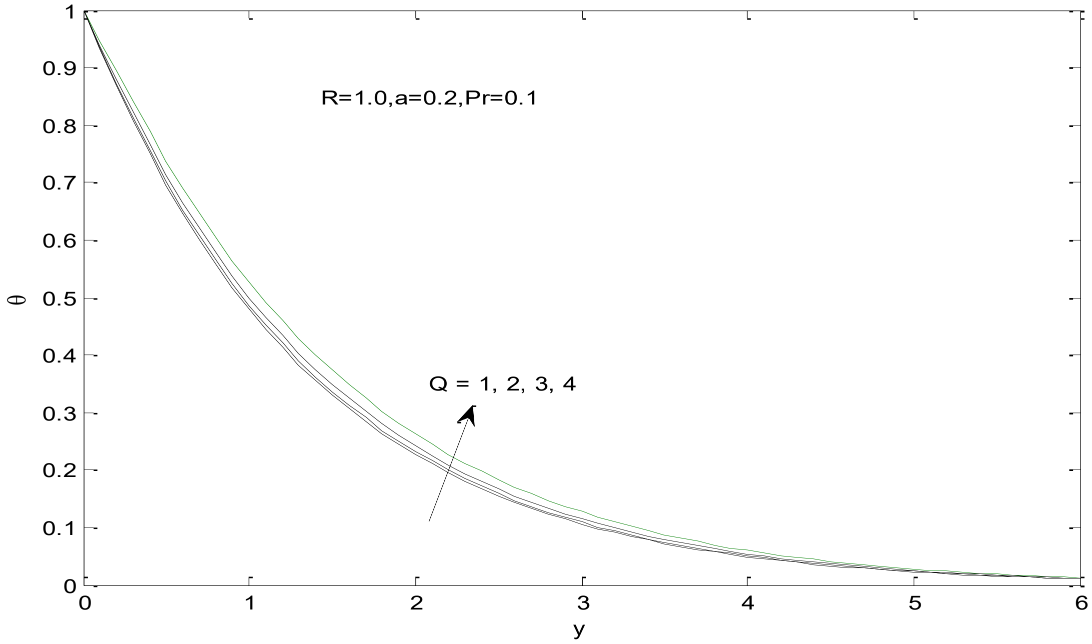

Figure (10): Temperature profiles for Q

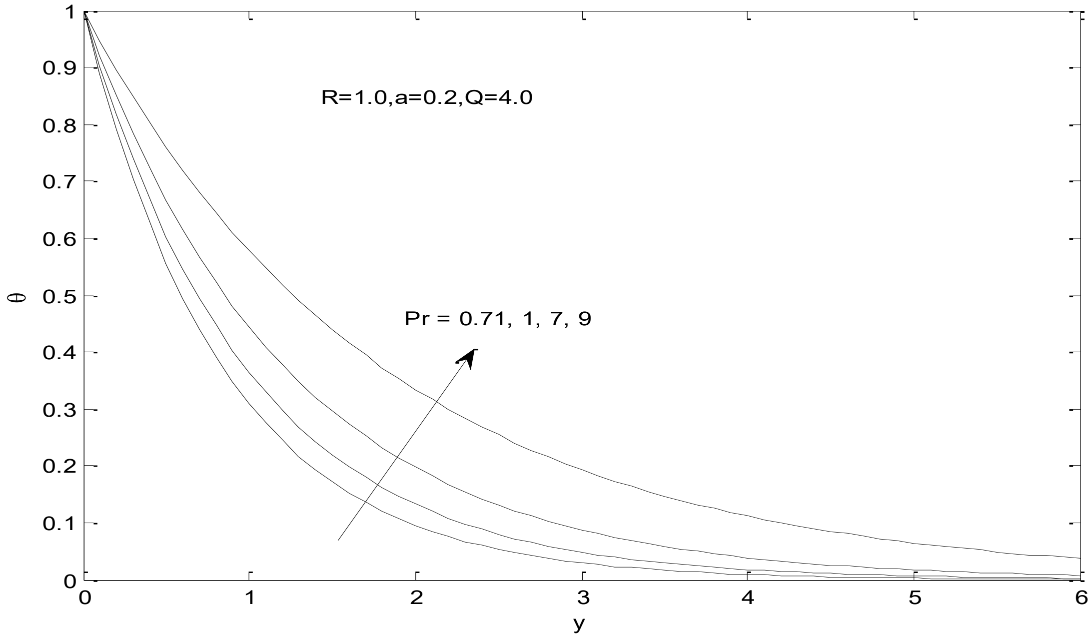

Figure (11): Temperature profiles for Pr

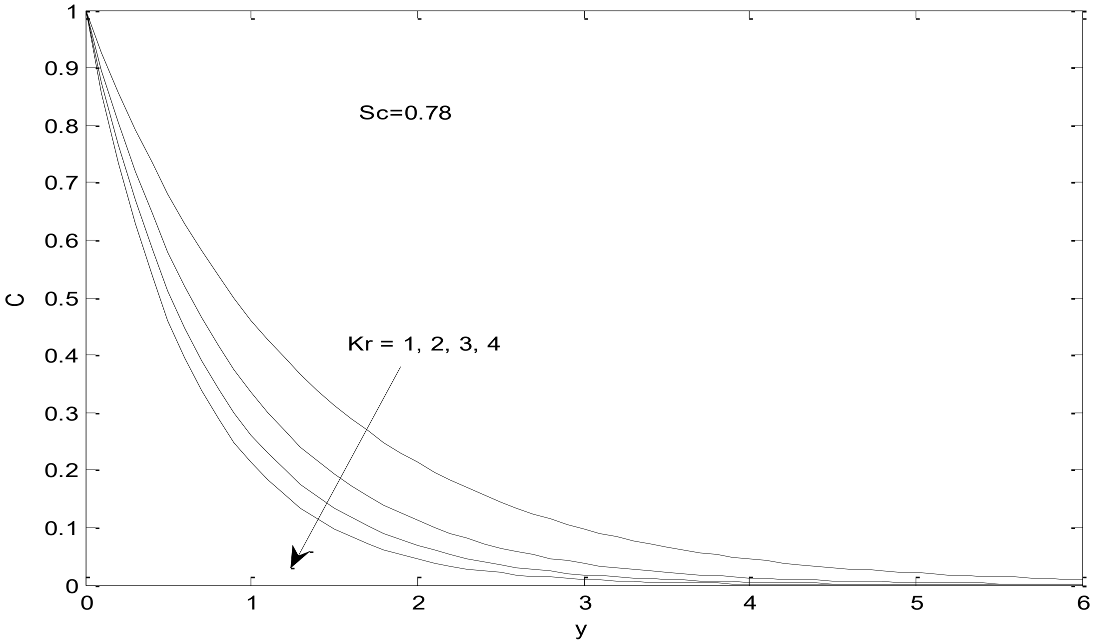

Figure (12): Concentration profiles for Kr

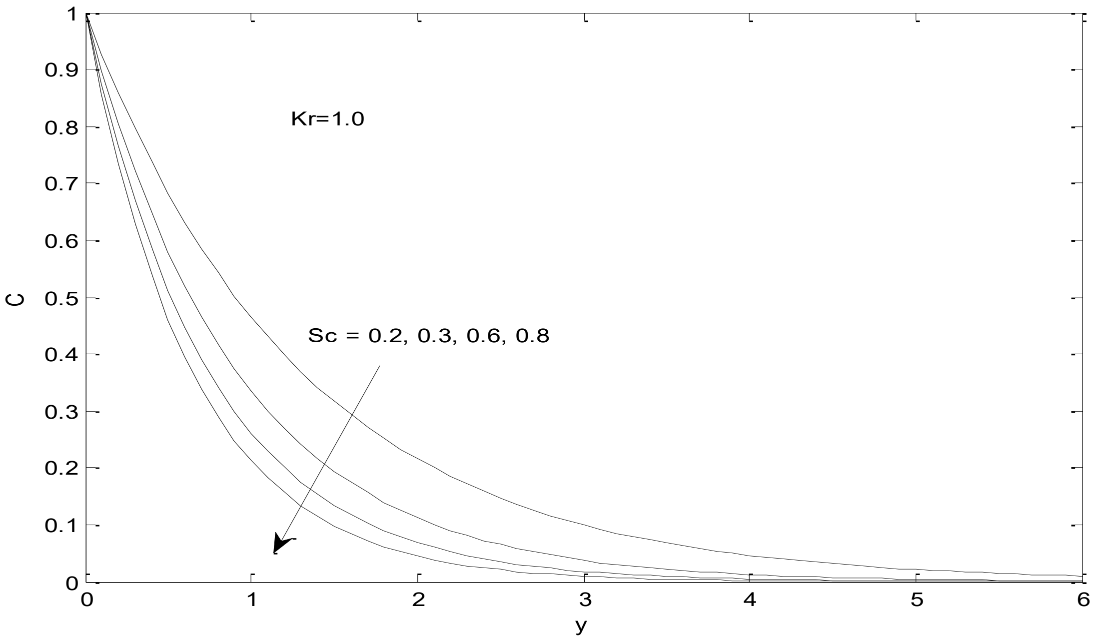

Figure (13): Concentration profiles for Sc